Time dependent correlations of inflationary perturbations

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Abstract. –
We show that if the primordial classical perturbations were generated by the gravitational particle creation during inflation, and followed by an evolution of quantum-to-classical transition, the time dependent correlation of these perturbations is long-tailed with a correlation time larger than the Hubble-time. Consequently, the inflationary perturbations are locally scale-scale correlated. Hence, the interaction of the fields during inflation can be explored via the detection of the local scale-scale correlation of the CMB fluctuations.

It is fundamentally important to understand the nature of the primordial cosmological perturbations which are responsible for the formation of structures that we see today. The inflation paradigm for the early universe assumes that the cosmic perturbations are initiated by the process of particle creation from vacuum in the background gravitational field of the expanding universe [1]. The subsequent decoherence of the quantum fluctuations leads to a quantum-to-classical transition [2], and gives rise to the initial perturbations in the radiation dominated era on scales beyond the Hubble radius. Therefore, the earliest evolution of the inflationary cosmological fluctuations is governed by the gravitational particle creation accompanying the quantum-to-classical transition. Recently, the mechanism of the gravitational particle creation is re-emphasized with the development of the quintessential inflation model [3]. However, it still seems to lack some testable predictions of these mechanisms.

In this *Letter*, we will show that the time-dependent correlation of the primordial perturbations is one of such testable predictions. In the "standard" inflation, i.e. the slow-roll inflation caused by a single scalar inflaton ϕ, the correlation time of the primordial perturbations is longer than the Hubble time H^{-1}. This correlation will entail a non-trivial observable feature – the local scale-scale correlation of the initial perturbations.

The time-dependence of cosmological perturbations is generally not observable in non-inflationary models, because the observed mass field does not contain two or multi-time information of the perturbations. In contrast, the inflationary scenario provides an excellent prospect to observing the time-dependence of the initial perturbations in a given region on the scale of horizon. Owing to the so-called "first out – last in" feature, a fluctuation crossing over the Hubble radius H^{-1} at the moment $t^{\times}$ will yield a perturbation at the end of the inflation, t_f, with the physical wavenumber given by [4]

$$k_p \propto e^{H(t^{\times}-t_f)}. \tag{1}$$

Consequently, the longer the physical length of the perturbations, the earlier the time of becoming the super-horizon scaled, and vice versa. Therefore, if perturbations with the same physical scale at two different instants $t_1^\times$ and $t_2^\times$ are correlated during the super-horizon evolution, the perturbations of the corresponding physical scales $k_p(t_1^\times)$ and $k_p(t_2^\times)$ at the same time t_f will be correlated. Equation (1) prescribes exactly the mapping of the correlation between two perturbations on the same physical scale with different horizon-crossing moments to that of two perturbations at the same time with different physical scales. The latter is actually observable. If the length scales of perturbations are larger than the Hubble radius H^{-1} at the decoupling era, the perturbations were less contaminated by the post inflationary reheating and other small scale processes. Thus, one can expect that the two-scale correlation of the cosmic background temperature fluctuations retain the information of the time dependent behavior of the initial perturbations.

The time-dependent correlation is not a new issue. It is generally believed that the perturbations originated from the quantum fluctuations of vacua during inflation have large correlation time since they vary slowly in the super-horizon regime. However, the rough idea of the lengthy correlation time is inadequate to establish the detectability of the time-dependent correlation , and one must study the temporal correlation of the perturbations in both quantum and classical regime to reach the equation suitable of calculating the time correlation in practical.

Let us calculate the time-dependent behavior for the "standard" slow-roll inflation governed by a real massless scalar field ϕ described by the action

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[\partial^\mu\phi\partial_\mu\phi - V(\phi)], \tag{2}$$

where $V(\phi)$ is a self-interaction potential. Under the slow-roll condition, the time-scale of the self-interaction is much longer than $1/H$. That is, the self-interaction is negligible if we study only the evolution on time scales smaller than the duration of inflation. Without interaction, the evolution of perturbations of the free ϕ field mode will be coherent. Thus, the ϕ field self-interaction is ineffective of violating the temporal coherence during inflation.

For the simple model Eq. (2), the dominant interaction during inflation is the coupling between the ϕ field and the gravitation of the expanding universe, governed by the gravitational scalar density $\sqrt{-g}$ in the integral. Therefore, in the "standard" inflation model, the origin and evolution of the temporal coherence of the primordial perturbations are actually determined by the dynamics of the gravitationally driven particle creation.

In a de Sitter space, the scalar field $\phi(\mathbf{r},t)$ can be described as a superposition of free modes with coordinate and conjugate momentum variables given by $\phi(\mathbf{k}) = (2\pi)^{-3/2}\int\phi(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{r}$ and $\pi(\mathbf{k}) = a^2\phi'(\mathbf{k})$, respectively, where $\mathbf{k}$ is the comoving wavevector and $a = e^{Ht}$ represents the cosmic scale factor [5]. The notation $'$ denotes the derivative with respect to the conformal time $\tau = \int dt/a(t) = -(1/H)e^{-Ht}$. Therefore, the time-dependent behavior of the ϕ field can be studied via the modes $\mathbf{k}$.

The quantum nature of the ϕ field is specified by the equal-time commutation relation given by $[\phi(\mathbf{k},\tau), \pi^\dagger(\mathbf{k},\tau)] = i\delta^{(3)}(\mathbf{k}-\mathbf{k}')$, which yields

$$\begin{aligned}\phi(\mathbf{k},\tau) &= \sigma_\mathbf{k}(\tau)a_\mathbf{k} + \sigma^*_\mathbf{k}(\tau)a^\dagger_{-\mathbf{k}}, \\ \pi(\mathbf{k},\tau) &= a^2\sigma'_\mathbf{k}(\tau)a_\mathbf{k} + a^2\sigma^{*\prime}_\mathbf{k}(\tau)a^\dagger_{-\mathbf{k}}.\end{aligned} \tag{3}$$

The $a_\mathbf{k}$ and $a^\dagger_\mathbf{k}$ are respectively, the annihilation and creation operators, satisfying commuta-

tion relations $[a_{\mathbf{k}}, a^{\dagger}_{\mathbf{k}'}] = \delta^{(3)}(\mathbf{k}-\mathbf{k}')$. The time dependence of the mode $\mathbf{k}$ is given by

$$\sigma_{\mathbf{k}}(\tau) = \frac{1}{\sqrt{2k}}\left[H\tau - i\frac{H}{k}\right]e^{-ik\tau}, \tag{4}$$

If the system is in the vacuum state at a given time τ_0, we have

$$a_{\mathbf{k}}|0,\tau_0\rangle = 0 \tag{5}$$

for all $\mathbf{k}$. In the Schrödinger representation, the evolution of the system leads the vacuum state at τ_0 developing into $|0,\tau\rangle = S|0,\tau_0\rangle$ at a later time τ, where S is the S-matrix. The state $|0,\tau\rangle$ is no longer a vacuum state. The average number of the particles gravitationally created in mode $\mathbf{k}$ is $N_{\mathbf{k}}(\tau) = \langle 0,\tau_0|S^{-1}a^{\dagger}_{\mathbf{k}}a_{-\mathbf{k}}S|0,\tau_0\rangle$.

One cannot directly calculate the time-dependent correlation of classical density perturbations by the state $|0,\tau\rangle$, because at a different τ the states are quantum coherent. We should first find the condition of quantum decoherence for the system at different times. In the coordinate representation $\phi(\mathbf{k})$, the Schrödinger wave function of the state $|0,\tau\rangle$ is given by the product of the wave functions for all $\mathbf{k}$ modes [6]:

$$\Psi[\phi(\mathbf{k}),\tau] = \frac{1}{\sqrt{\pi}|\sigma_{\mathbf{k}}(\tau)|}\exp\left\{-\frac{|\phi(\mathbf{k})|^2}{2|\sigma_{\mathbf{k}}(\tau)|^2}[1-i2F(\mathbf{k},\tau)]\right\}, \tag{6}$$

where

$$F(\mathbf{k},\tau) = \frac{1}{2k\tau} = -\frac{H}{2k}e^{Ht}. \tag{7}$$

Equation (6) is actually the ground state wave function of a harmonic oscillator with coordinate $|\phi(\mathbf{k})|$ which possesses a time-dependent variance in itself.

Thus, the quantum coherence between the coordinates at different times can be calculated as

$$\langle\Psi(\tau_2)|\phi^{\dagger}(\mathbf{k})\phi(\mathbf{k})|\Psi(\tau_1)\rangle = \frac{1}{2\sqrt{\pi}|\sigma_{\mathbf{k}}(\tau_1)||\sigma_{\mathbf{k}}(\tau_2)|} \left\{\frac{1}{2|\sigma_{\mathbf{k}}(\tau_1)|^2}[1-i2F(\mathbf{k},\tau_1)] + \frac{1}{2|\sigma_{\mathbf{k}}(\tau_2)|^2}[1+i2F(\mathbf{k},\tau_2)]\right\}^{-3/2}. \tag{8}$$

When $t \gg H^{-1}$ or $|H\tau| = e^{-Ht} \ll 1$, the term $H\tau$ in Eq. (4) can be ignored, and $|\sigma_{\mathbf{k}}(\tau)|$ becomes τ-independent. Accordingly, $|\sigma_{\mathbf{k}}(\tau_1)| \simeq |\sigma_{\mathbf{k}}(\tau_2)|$, and the ratio of the two-time matrix element to the diagonal element reduces to

$$\frac{|\langle\Psi(\tau_2)|\phi^*(\mathbf{k})\phi(\mathbf{k})|\Psi(\tau_1)\rangle|^2}{|\langle\Psi(\tau_1)|\phi^*(\mathbf{k})\phi(\mathbf{k})|\Psi(\tau_1)\rangle|^2} \simeq \left\{1+\frac{1}{4}\left(\frac{H}{k}\right)^2\left[e^{Ht_1}-e^{Ht_2}\right]^2\right\}^{-3/2}. \tag{9}$$

Evidently, Eq. (9) is quickly approaching zero which implies that the decoherence of the system for all $|t_1-t_2| \geq 1/H$, once ke^{-Ht_1} and ke^{-Ht_2} are less than H. Thus, the relationship between the $\mathbf{k}$-mode fluctuations at times t_1 and t_2 can be treated as classical perturbations provided that both physical wavelengths, e^{Ht_1}/k and e^{Ht_2}/k, are super-Hubble sized with the time separation $|t_1-t_2|$ being greater than one Hubble time.

The quantum decoherence does not exclude the classical time correlation of perturbations [7]. The classical correlation can be derived by means of the Wigner distribution functions of

the system. For the $\mathbf{k}$ mode, it reads

$$\begin{aligned} W(\phi(\mathbf{k}),\pi(\mathbf{k}),\tau) &= \int d\left(\frac{\varphi}{2\pi}\right) e^{-i\pi\varphi}\Psi^*\left(\phi(\mathbf{k})-\frac{\varphi}{2},\tau\right)\Psi\left(\phi(\mathbf{k})+\frac{\varphi}{2},\tau\right) \\ &= N_{\mathbf{k}}\frac{|\sigma_{\mathbf{k}}|^2}{\pi}\exp\left[-\frac{|\phi(\mathbf{k})|^2}{|\sigma_{\mathbf{k}}(\tau)|^2}\right]\exp\left[-|\sigma_{\mathbf{k}}(\tau)|^2\left|\pi(\mathbf{k})-\frac{F(\mathbf{k},\tau)}{|\sigma_{\mathbf{k}}(\tau)|^2}\phi(\mathbf{k})\right|^2\right] \end{aligned} \quad (10)$$

Since $|\sigma_{\mathbf{k}}(\tau)|$ is τ-independent when $t \gg 1/H$, the first exponent on the right hand side of Eq.(10) represents the Gaussian probability distribution of the fluctuation of the scalar field $\phi(\mathbf{k})$.

The time-dependent behavior of the classical perturbations can be extracted through the second Gaussian probability distribution in Eq. (10). In the classical limit $\hbar \longrightarrow 0$, it yields

$$W(\phi(\mathbf{k}),\pi(\mathbf{k}),\tau) \simeq N_{\mathbf{k}}\exp\left[-\frac{|\phi(\mathbf{k})|^2}{|\sigma_{\mathbf{k}}(\tau)|^2}\right]\delta\left(\pi(\mathbf{k})-\frac{F(\mathbf{k},\tau)}{|\sigma_{\mathbf{k}}(\tau)|^2}\phi(\mathbf{k})\right). \quad (11)$$

This shows that the classical trajectory of the $\mathbf{k}$ mode in phase space is given by

$$\pi(\mathbf{k})-\frac{F(\mathbf{k},\tau)}{|\sigma_{\mathbf{k}}(\tau)|^2}\phi(\mathbf{k})=0. \quad (12)$$

The degree, or the effectiveness, of classical correlation can be measured by the relative sharpness of the classical trajectory in phase space, which is defined as the ratio of the dispersion in momentum to the magnitude of the average of the momentum $|\pi(\mathbf{k})|$ [8]. By virtue of Eq. (10), this ratio is

$$\frac{1/|\sigma_{\mathbf{k}}(\tau)|}{|(F(\mathbf{k},\tau)/|\sigma_{\mathbf{k}}(\tau)|^2)\phi(\mathbf{k})|} \simeq \frac{1}{|F(\mathbf{k},\tau)|} \ll 1, \quad \text{if } |k\tau| = (k/H)\exp(-Ht) \ll 1. \quad (13)$$

Therefore, the classical perturbations of super-Hubble size, i.e. $ke^{-Ht} \ll H$, are perfectly coherent. Namely, after the quantum-to-classical transition, the evolution of the initial perturbations can be calculated by the classical trajectory Eq. (12).

The scale t_c of the time-dependent correlation between classical perturbations of $\mathbf{k}$ mode can be characterized by

$$\frac{1}{t_c} \simeq -\frac{\partial\phi(\mathbf{k})/\partial t}{\phi(\mathbf{k})} = -\frac{\partial\tau}{\partial t}\frac{\pi(\mathbf{k})}{a^2\phi(\mathbf{k})} \simeq \frac{k^2e^{-2Ht}}{H}. \quad (14)$$

Hence, for super-Hubble sized perturbations, we have

$$t_cH \gg 1. \quad (15)$$

That is, the correlation time of the primeval perturbations is much longer than the Hubble time H^{-1}.

This result is anticipated because the only interaction of the inflation in the "standard" model during inflation is the gravitational particle creation. After the quantum-to-classical transition, the perturbed scalar field consists of classical free motion. Since there is no coupling between either comoving modes $\mathbf{k}$ or physical modes $\mathbf{k}_p$, the evolution of the classical perturbations of the ϕ field are coherent. Therefore, one may conclude that the large correlation time of the initial perturbations probably is a generic feature for models with free inflaton, i.e., besides the gravitational coupling of the inflaton, there are no interactions and

self-interaction during the slow-roll phase. This point can be seen more clearly if we establish an equation for the classical perturbations of the ϕ field.

Actually, one can regard Eq.(14) as an evolution equation of the ensemble averaged $\phi(\mathbf{k})$, i.e. $\partial\phi(\mathbf{k})/\partial t \simeq -(k^2e^{-2Ht}/H)\phi(\mathbf{k})$. The uncertainty of this equation can be estimated by the time scale given by

$$\frac{1}{t'_c} \simeq \left|\frac{\Delta\partial\phi(\mathbf{k})/\partial t}{\phi(\mathbf{k})}\right| = \frac{|\Delta\pi(\mathbf{k})|}{a^3\,|\phi(\mathbf{k})|} \simeq \frac{k^3e^{-3Ht}}{H^2}. \tag{16}$$

We have $t'_c \gg t_c$ and therefore, the classical evolution equation for the $\mathbf{k}$ mode perturbations in phase space is reasonable in average.

The two equations (14) and (16) of the ensemble averaged mode $\phi(\mathbf{k})$ can be combined into the following Langevin-like equation for stochastic variable $\phi(\mathbf{k})$

$$\frac{\partial\phi(\mathbf{k})}{\partial t} \simeq -\frac{k^2e^{-2Ht}}{H}\phi(\mathbf{k}) + \eta_{\mathbf{k}}, \tag{17}$$

where the small Gaussian noise $\eta_{\mathbf{k}}$ has zero mean with variance $\simeq k^{3/2}e^{-3Ht}/H$. Therefore, the ensemble average of (17) yields Eq.(14), and the variance gives (16). Since Eq.(17) is linear to $\phi(\mathbf{k},t)$, it is straightforward to get the equation of $\phi(\mathbf{r},t)$ in the comoving $\mathbf{r}$-representation. After employing the physical coordinates in which $dx_i = e^{Ht}dr_i$ $(i = 1, 2, 3)$, Eq.(17) becomes

$$\frac{\partial\phi(\mathbf{x},t)}{\partial t} \simeq \frac{1}{H}\nabla^2_{\mathbf{x}}\phi(\mathbf{x},t) + \eta(\mathbf{x},t), \tag{18}$$

where $\nabla^2_{\mathbf{x}}$ stands for the Laplacian of the physical coordinates $\mathbf{x}$. Because the variance of the stochastic noise is scaled as $\langle|\eta(\mathbf{x},t)|^2\rangle \propto e^{-3Ht}$, the noise term is strongly suppressed as $Ht \gg 1$.

Equation (18) shows that the correlation time for a mode with the physical wavevector $\mathbf{k}_p$, i.e. $\phi(\mathbf{x},t) \sim e^{-i\mathbf{k}_p\cdot\mathbf{x}}$, is proportional to k_p^{-2}. This is typical to the diffusion-like soft modes in the theory for non-equilibrium systems [9]. If a perturbed field consists of the superposition of such soft long wavelength perturbations described by Eq.(18), the time dependent correlation function of the perturbed ϕ field possesses a long-tail [10]. This behavior is more transparent when considering modes characterized by $e^{-i\omega t - i\mathbf{k}_p\cdot\mathbf{x}}$. For these modes Eq.(18) gives rise to the dispersion relation as $\omega = -ik_p^2/H$. Apparently, the relaxation time $-1/\Im\omega_{\mathbf{k}_p} \to \infty$ when $k_p \to 0$, and these modes lead to long-range correlations in general.

Since we are only interested in the perturbations consisting of modes with physical wavelength larger than H^{-1}, the field $\phi(\mathbf{x},t)$ crossing over the horizon can be prescribed by

$$\phi(\mathbf{x},t) = \int_{k_p<H} d^3\mathbf{k}_p\hat{\phi}_{\mathbf{k}_p}e^{-i\omega t - i\mathbf{k}_p\cdot\mathbf{x}}, \tag{19}$$

where $\hat{\phi}_{\mathbf{k}_p}$ is the amplitude of the perturbation for modes $\mathbf{k}_p$. Thus, the time-dependent correlation function is determined by

$$\langle\phi(\mathbf{x},t_1)\phi^*(\mathbf{x},t_2)\rangle = \int_{k_p<H} d^3\mathbf{k}_p\langle\hat{\phi}_{\mathbf{k}_p}\hat{\phi}^*_{\mathbf{k}_p}\rangle \exp\left(-\frac{k_p^2}{H}|t_1 - t_2|\right). \tag{20}$$

At a given instant, the distribution of $\langle\hat{\phi}\hat{\phi}^*\rangle$ with respect to the $\mathbf{k}_p$ modes should be the same as that along the $\mathbf{k}$ modes, since the relation between $\mathbf{k}_p$ and $\mathbf{k}$ is specified by a constant

multiplier. On the other hand, Eqs.(4) and (11) imply that, $\langle|\phi(\mathbf{k})|^2\rangle \simeq |\sigma_{\mathbf{k}}(\tau)|^2 \propto k^{-3}$ in the super-horizon region. Therefore, $\langle\hat{\phi}_{\mathbf{k}_p}\hat{\phi}^*_{\mathbf{k}_p}\rangle \propto k_p^{-3}$, and the integral (20) is infrared divergent. However, the duration of the inflation spans only a finite period of time, there must be an infrared cutoff at $k \simeq H/R$ with $R \gg 1$. Thus, $\langle\hat{\phi}_{\mathbf{k}_p}\hat{\phi}^*_{\mathbf{k}_p}\rangle$ can be approximated as a quantity independent of k_p, and Eq.(20) yields

$$\langle\phi(\mathbf{x},t_1)\phi^*(\mathbf{x},t_2)\rangle \propto \left(\frac{t_c}{|t_1-t_2|}\right)^{3/2}, \quad \text{if} \quad |t_1-t_2| > t_c, \tag{21}$$

where the correlation time $t_c \simeq C/H$, and the constant $C > 1$, i.e. the correlation time is longer than the Hubble time. Hence, during the super-Hubble evolution, the field ϕ is coherent regardless of the horizon-crossing moments of the perturbations provided that time difference is less than C/H. This result illuminates that the gravitational particle creation can be considered as a dissipation source for the non-equilibrium inflaton ϕ during the slow-roll. The relaxation of that dissipative process is dominated by soft modes and subsequently gives rise to the classical perturbations with a long-tailed time correlation. On the other hand, the soft modes do not induce any long range spatial correlation and therefore, the perturbations in different spaces $\mathbf{x}$ retain themselves. That is, the random field of the inflationary perturbations is "double-faced": its spatial distribution is Gaussian, while the temporal or scaled distribution is coherent. This property leads to a measurable effect if we use field variables based on a space-scale decomposition.

Let us consider such space-scale decomposition given by the bases $\Psi_{k_p,x}(\mathbf{x}')$ which are localized, orthogonal and complete. The indices k_p and x denote a cell in the phase (x, k_p) space, i.e. $k_p \to k_p + \Delta k_p$, $x \to x + \Delta x$, and the differential volume element $\Delta x \Delta k_p \simeq 2\pi$. Wavelet analysis provides various bases for this sort of space-scale decomposition [11]. With a suitable wavelet decomposition, the ϕ field can be described by the variables $\delta\phi_{k_p,x}$ defined as

$$\delta\phi_{k_p,x} = \int \phi(\mathbf{x}')\Psi_{k_p,x}(\mathbf{x}')\mathbf{dx}'. \tag{22}$$

Obviously, $\delta\phi_{k_p,x}$ represents the amplitude of the perturbation of mode (k_p, x), i.e. the fluctuations of the field at position around x and on scale around k_p.

According to Eq.(1), the amplitude $\delta\phi_{k_p,x}$ is mainly attributed to $\phi(\mathbf{x})$ with the horizon-crossing time corresponding to k_p. Consequently, the time-time correlation of Eq.(21) implies

$$\langle\delta\phi_{k_{p1},x}\delta\phi_{k_{p2},x}\rangle \neq 0, \tag{23}$$

if the difference between the horizon-crossing times corresponding to k_{p1} and k_{p2} is less than C/H. Equation (23) is a local (at spatial area around $\mathbf{x}$) scale-scale (k_{p1} and k_{p2}) correlation.

By means of the variables $\delta\phi_{k_p,x}$ one can easily perceive this "double-faces" feature of the inflationary perturbations. The variables $\delta\phi_{k_p,x}$ which are Gaussian with respect to x entail

$$\langle\delta\phi_{k_p,x}\delta\phi_{k_p,x'}\rangle = P(k_p)\delta_{\mathbf{x},\mathbf{x}'}, \tag{24}$$

and all higher order cumulants of $\delta\phi_{k_p,x}$ are zero, in which $P(k_p)$ is the power spectrum of the Gaussian fluctuations $\delta\phi_{k_p,x}$ [12]. Meanwhile, the variables at different scales k_p may have carried some correlation. As a simple example, let us consider

$$\delta\phi_{k'_p,x} = \alpha\delta\phi_{k_p,x}, \tag{25}$$

where α is a constant. In this case, the perturbation $\delta\phi_{k'_p,x}$ with respect to x is also Gaussian, i.e. its variance is $P(k'_p) = \alpha^2 P(k_p)$ and all higher order cumulants of $\delta\phi_{k'_p,x}$ are zero. However,

the scale-scale (k_p'-k_p) correlation at the place is significant, i.e. $\langle \delta\phi_{k_p',x}\delta\phi_{k_p,x}\rangle \propto \alpha$. Therefore, a Gaussian power spectrum $P(k_p)$ can coexist with a local scale-scale correlation [Eq.(23)]. It should be emphasized that as a statistical measure, the local scale-scale correlation is independent of the Gaussian power spectrum. One cannot determine whether there is and/or how strong it is the local scale-scale correlation directly by their power spectrum.

Recently, the search for the local scale-scale correlation of large scale structure samples has attracted attention. Using the wavelet technique, the local scale-scale correlations of QSO Lyα forests [13] and COBE data [14] have been studied. On the other hand, the time-dependent correlation of the primordial perturbations is sensitive to the interaction of the inflationary field. It is also sensitive to dissipation during the inflation. For instance, if the cosmic inflation is thermally dissipated [15], the correlation time of the initial cosmic perturbations will be significantly changed by a thermal-dissipative term in Eq.(17). Hence, the local scale-scale correlation behavior of the CMB fluctuations and other relevant samples would be useful to discriminate among models of inflation.

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